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Abgabe: Keine!

Übungen zur Vorlesung
Methoden des Algorithmenentwurfs
SS 2017
Blatt 11

Aufgabe 29:
3-Coloring is a yes/no question, but we can phrase it as an optimization problem as follows: Suppose we are given a graph $G = (V, E)$ and we want to color each node with one of three colors, even if we are not necessarily able to give different colors to every pair of adjacent nodes. Rather, we say that an edge (u, v) is *satisfied* if the colors assigned to u and v are different.
Consider a 3-Coloring that maximizes the number of satisfied edges, and let $c^\star$ denote this number. Give a randomized polynomial-time algorithm that produces a 3-coloring where the expected number of edges it satisfies should be at least $\frac{2}{3}c^\star$.

Aufgabe 30:
Consider a country in which 100.000 people vote in an election. There are only two candidates on the ballot: a Democratic candidate (denoted D) and a Republican candidate (denoted R). As it happens, this country is heavily Democratic, so 80.000 people go to the polls with the intention of voting for D, and 20.000 go to polls with the intention of voting for R.
However, the layout of the ballot is a little confusing, so each voter, independently and with probability $\frac{1}{100}$, votes for the wrong candidate – that is, the one that he or she did not intend to vote for.
Let X denote the random variable equal to the number of votes received by the Democratic candidate D, when the voting is conducted with this process of error. Determine the expected value of X, and give an explanation of your derivation of this value.

Aufgabe 31:
A number of *peer-to-peer systems* on the Internet are based on *overlay networks*. Rather than using the physical Internet topoloy as the network on which to perform computation, these systems run protocols by which nodes choose collections of virtual „neighbors“ so as to define a higher-level graph whose structure may bear little or no relation to the underlying physical network. Such an overlay network is then used for sharing data and services, and it can be extremely flexible compared with a physical network, which is hard to modify in real time to adapt to changing conditions.
Peer-to-peer networks tend to grow through the arrival of new participants, who join by linking into the existing structure. This growth process has an intrinsic effect on the characteristics of the overall network. Recently, people have investigated simple abstract models for networks growth that might provide insight into the way such processes behave, at a qualitative level, in real networks.
Here is a simple example of such a model. The system begins with a single node v_1. Nodes then join one at a time; as each node joins, it executes a protocol whereby it forms a directed

link to a single other node chosen uniformly at random from those already in the system. More concretely, if the system already contains nodes $v_1, v_2, \ldots, v_{k-1}$ and node v_k wishes to join, it randomly selects one of $v_1, v_2, \ldots, v_{k-1}$ and links to this node.
Suppose we run this process until we have a system consisting of nodes $v_1, v_2, \ldots, v_n$; the random process described above will produce a directed network in which each node other than v_1 has exactly one outgoing edge. On the other hand, a node may have multiple incoming links, or none at all. The incoming links to a node v_j reflect all the other nodes whose access into the system is via v_j; so if v_j has many incoming links, this can place a large load on it. To keep the system loadbalanced, then, we would like all nodes to have a roughly comparable number of incoming links. That is unlikely to happen here, however, since nodes that join earlier in the process are likely to have more incoming links than nodes that join later. Let us try to quantify this imbalance as follows:

a) Given the random process described above, what is the expected number of incoming links to node v_j in the resulting network? Give an exact formula in terms of n and j, and also try to express this quantity asymptotically T (via an expression without large summations) using $\Theta(\cdot)$ notation.

b) Part(a) makes precise a sense in which the nodes that arrive early carry an „unfair“ share of the connections in the network. Another way to quantify the imbalance is to observe that, in a run of this random process, we expect many nodes to end up with no incoming links. Give a formula for the expected number of nodes with no incoming links in a network grown randomly according to this model.