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Abgabe: Keine!

Übungen zur Vorlesung

Methoden des Algorithmenentwurfs

SS 2017

Blatt 9

Aufgabe 22:
Consider the *Center Selection Problem* where we are given a set of sites $S = \{s_1, \dots, s_n\}$ in the plane, and we want to choose a set of k centers $C = \{c_1, \dots, c_k\}$ whose covering radius (i.e. the farthest that people in any one site must travel to their nearest center) is as small as possible.
Moreover, consider the following local search algorithm for the Center Selection Problem: We start by arbitrarily choosing k points in the plane to be the centers $c_1, \dots, c_k$. We now alternate the following steps:

1. Given the set of k centers $c_1, \dots, c_k$, we divide S into k sets: For $i = 1, \dots, k$, we define S_i to be the set of all the sites for which c_i is the closest center.

2. Given the division of S into k sets, construct new centers that will be as central as possible relative to them. For each set S_i, we find the smallest circle in the plane that contains all points in S_i, and define center c_i to be the center of this circle.

If steps 1. and 2. cause the covering radius to strictly decrease, then we perform another iteration; otherwise the algorithm stops.

a) Prove that this local search algorithm terminates.

b) Prove or disprove the following statement: There is an absolute constant $b > 1$, so when the local search algorithm terminates, the covering radius of its solution is at most b times the optimal covering radius.

Aufgabe 23:
Consider the problem of finding a stable state in a Hopfield neural network, in the special case when all edge weights are positive. Now suppose the underlying graph $G = (X \cup Y, E)$ is connected and bipartite. Then there is a natural „best“ configuration for the Hopfield net, in which all nodes in X have the state $+1$ and all nodes in Y have the state -1.
Prove or disprove the following statement: In this special case, the *State-Flipping Algorithm* described in the lecture always find this „best“ configuration.

Aufgabe 24:
Recall that a *matching* in a graph $G = (V, E)$ is a set of edges $M \subseteq E$ with the property that each node appears in at most one edge of M. Now, consider the following *Hill Climbing Algorithm* for finding a matching in a bipartite graph: *As long as there is an edge whose endpoints are unmatched, add it to the current matching. When there is no longer such an edge, terminate with a locally optimal matching.*

a) Give an example of a bipartite graph G for which this hill climbing algorithm does not return the maximum matching.

b) Let M and M' be matchings in a bipartite graph G. Suppose $|M'| > 2|M|$. Show that there is an edge $e' \in M'$ such that $M \cup \{e'\}$ is a matching in G.

c) Use b) to conclude that any locally optimal matching returned by the hill climbing algorithm in a bipartite graph G is at least half as large as a maximum matching in G.