

**Randomized Algorithms**  
SS 2018  
**Homework Assignment 10**

**Problem 27:**

In the shortest pairwise distance problem we are given a set of points  $V$  in the 2-dimensional Euclidean space and the problem is to find the pair of points of shortest Euclidean distance in  $V$ . Show that this is an LP-type problem and determine its combinatorial dimension.

**Problem 28:**

In the largest included rectangle problem we are given an arbitrary polygon  $P$  in a 2-dimensional Euclidean space that is specified by a sequence of corners  $V_P = \{v_1, \dots, v_n\}$  and the goal is to find a rectangle of largest volume that can be placed inside of  $P$ . Show that this is an LP-type problem and determine its combinatorial dimension.

**Problem 29:**

Consider any integer linear program  $P$  with objective function  $f(x) = c^T \cdot x$  and constraints  $Ax \leq b$  that has a finite number of solutions. Let  $\#P$  be the problem of counting the number of feasible solutions for  $P$ , i.e., the number of vectors  $x \in \mathbb{Z}^n$  that satisfy  $Ax \leq b$ . Show that if  $\#P$  can be solved in polynomial time then the optimal solution of  $P$  can be found in polynomial time.

**Problem 30:**

Prove Theorem 8.3.