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Randomized Algorithms

SS 2018

Homework Assignment 4

In this homeowork assignment we are focusing on the problem of distributing n balls among n bins as evenly as possible. Various strategies have been considered for that. We will be investigating the following three strategies which consider the balls one after the other.

Problem 12:
RANDOM strategy: For each ball, pick a bin uniformly and independently at random.
Compute the expected number of balls in a bin and show that with high probability (i.e., with probability at least $1 - 1/n$) every bin has at most $c \log n / \log \log n$ many balls, for a sufficiently large constant c.
(Hint: First, show that the probability is at most $1/n^2$ that some fixed bin i has at least $c \log n / \log \log n$ many balls, and conclude from that that the probability is at most $1/n$ that there is a bin with at least $c \log n / \log \log n$ many balls.)

Problem 13:
THRESHOLD strategy: For each ball i, continue to pick a bin uniformly and independently at random till a bin is found with at most one ball.
Compute (an upper bound on) the expected number of times a bin is picked over all n balls.
(Hint: properly define a random variable for the number of attempts for each ball and determine its expected value.)
Is it possible to apply Chernoff bounds to show that, summed up over all balls, the expected number of times a bin is picked is $O(n)$ with high probability? If so, how should the binary random variables be defined?

Problem 14:
BALANCE strategy: For each ball i, pick two bins $b_1(i)$ and $b_2(i)$ uniformly and independently at random and initially place ball i in bin $b_1(i)$. As long as there is a bin b with more than 4 balls, pick a random ball in b and move it to its alternative bin.
It is known that this process terminates if there is no subset U of balls where all bin options land in some subset B of bins with $|B| \leq |U|/4$. Show that the probability is very high for this to be true.
(Hint: First, determine the probability that all bin options land in some subset B for some fixed subsets U and B with $|B| = |U|/4$. Use that to bound the probability that there is a subset U and a subset B with that property.)