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# Randomized Algorithms

## SS 2018

## Homework Assignment 6

**Problem 17:**

Let $G = (V, E)$ be a graph with vertex set $V = \{v_1, \ldots, v_n\}$. A *vertex cover* of $G$ is a set $C \subseteq V$ with the property that for every edge $\{v_i, v_j\} \in E$, either $v_i$ or $v_j$ is in $C$. In the VertexCover problem the task is to find the smallest possible vertex cover.

(a) Show that the following IP is an arithmetization of VC (i.e., there are transformations that transform a feasible solution of one problem formulation into a feasible solution of the other while maintaining the objective value):

Integer program for VC:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} x_i \\ \text{subject to} \quad & x_i + x_j \geq 1 \quad \text{for all } \{v_i, v_j\} \in E \\ & x_i \in \{0, 1\} \quad \text{for all } v_i \in V \end{aligned}$$

(b) Show that a deterministic rounding of the optimal solution of the linear relaxation of this IP according to the rule "if $x_i \geq 1/2$, then add $v_i$ to $C$" results in a feasible solution.

**Problem 18:**

Max2SAT is the restriction of the MaxSAT problem to Boolean formulas in CNF that have clauses with at most 2 literals. The decision variant of the Max2SAT problem is known to be NP-hard. Consider the following arithmetization of Max2SAT.

For each Boolean variable $x_i$ we define a variable $y_i$ that can take the value $-1$ or $+1$. In addition to that we have a variable $y_0 \in \{-1, +1\}$ with the meaning that $x_i$ is True if and only if $y_i$ and $y_0$ have the same value.

For the arithmetization of a clause $C$ with one literal, we distinguish between two cases:

- $C = x_i$: use $(1 + y_i \cdot y_0)/2$
- $C = \bar{x}_i$: use $(1 - y_i \cdot y_0)/2$

For clauses with two literals we can specify similar formulas.

1. Propose an arithmetization for all 4 possibilities for a clause with 2 literals.

2. Formulate a quadratic program for MAX2SAT with the help of these arithmetizations.
3. Formulate a semidefinite program as a relaxation of the quadratic program.
4. Propose an approximation algorithm for MAX2SAT. Do you have an idea how to show that its approximation ratio is at most 1.139?