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Advanced Distributed Algorithms and Data Structures

SS 2019

Homework Assignment 11

Problem 1:
Let $X = \{1, \ldots, n\}$ be a set of elements and $\mathcal{S}$ be a collection of subsets of X. A subset $H \subseteq X$ is called a *hitting set* of $\mathcal{S}$ if for all $S \in \mathcal{S}$, $S \cap H \neq \emptyset$. In the *hitting set problem* we are given $(X, \mathcal{S})$, and the goal is to find a hitting set of minimum size. Explain why (X, f), where $f(U)$ for any subset U of X denotes the number of sets in $\mathcal{S}$ intersected by U, satisfies the monotonicity and locality conditions (which implies that the hitting set problem is an LP-type problem).

Problem 2:
Suppose that the maximum size of a basis is much larger than the maximum size of an *optimal* basis. Would we be able to exploit that in order to obtain a better bound on the number of repeat-iterations than $O(dim(H, f) \cdot \log n)$? What about the sample size r?

Problem 3:
Show that in general, a bound of $O(d \log n)$ on the number of repeat-iterations is best possible. Hint: Assume that d is a constant and that there is a unique optimal basis B of H. How likely is it that R contains B?