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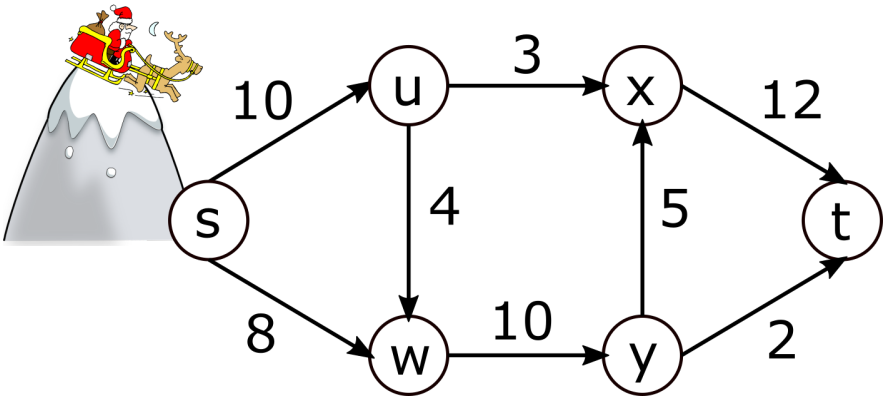
Fundamental Algorithms

WS 2017

Exercise Sheet 11

Christmas Edition

Exercise 1:
Santa wants to build the perfect sled run. The track starts at position s and ends at t. The intermediate hills of the track are connected by slopes. Santa's job is to put as much snow on the slopes as possible, but, for obscure reasons, each slope has limited snow capacity and the total snow raised towards an intermediate hill must eventually equal its outgoing snow. Therefore, we model the problem as a maximum flow problem. The corresponding flow network is given below.



Execute Goldberg's algorithm to help Santa build the perfect track. For each iteration of the while-loop, specify the executed operation (i.e., push snow down a slope or increase the height of a hill) and give the resulting preflow and the heights of the hills.

Exercise 2:
Let (G, s, t, c) be a flow network with integer capacities. Suppose that we are given a maximum flow in G.

a) Suppose we increase the capacity of a single edge $(u, v) \in E$ by 1. Give an $O(|V|+|E|)$-time algorithm to update the maximum flow.

b) Suppose we decrease the capacity of a single edge $(u, v) \in E$ by 1. Give an $O(|V|+|E|)$-time algorithm to update the maximum flow.

Exercise 3:
After their work has been done, the little elves $E_1, \ldots, E_r$ living in Santa's Wonderland spend the rest of the year discussing christmas-political issues and defending their labor rights. Each elve is member of at most one of the political parties $P_1, \ldots, P_p$ and enlisted in at least one of the labor unions $L_1, \ldots, L_q$. Each labor union must nominate one of its members to represent it on the christmas council so that the number of council members belonging to the political party P_k is at most u_k. Each elve can only be nominated by at most one labor union. Santa wants to determine whether there is a council that satisfies this "balancing" property. Help him decide this question by modeling the problem as a maximum flow problem.

Exercise 4:
Let (G, s, t, c) be a flow network. Give a polynomial-time algorithm to decide whether G has a *unique* minimum cut (S, T) (i.e., a cut of capacity strictly less than the capacity of all other cuts).

We wish everybody a Merry Christmas and a Happy New Year!