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Fundamental Algorithms

WS 2017

Exercise Sheet 12

Exercise 1:
Let (G, s, t, c) be a flow network with capacity function $c : V \times V \to \mathbb{N}_0$. Prove or disprove the following claims:

a) If $c(e)$ is even for all $e \in E$, then there exists a maximum flow in G that contains only even flow values.

b) If $c(e)$ is odd for all $e \in E$, then there exists a maximum flow in G that contains only odd flow values.

Exercise 2:
Let (G, s, t, c) be a flow network. An edge (u, v) is called *most vital edge*, if its deletion from E causes the largest decrease (among all edges) in the maximum flow value of (G, s, t, c). Similarly, a *least vital edge* is an edge whose deletion causes the least decrease in the maximum flow value. Prove or disprove the following claims:

a) A most vital edge is an edge with maximum capacity.

b) A most vital edge is an edge with maximum flow value (among all edges) given any maximum flow function.

c) A most vital edge is an edge of a minimum cut with maximum flow value in any maximum flow.

d) An edge that does not belong to some minimum cut cannot be a most vital edge.

e) A network might contain several most vital edges.

f) Any edge $(u, v) \in E$ with $f(u, v) = 0$ in any maximum flow function f is a least vital edge.

g) A least vital edge is an edge with minimum flow value $f(u, v)$ in any maximum flow function f.

h) Any edge in a minimum cut cannot be a least vital edge.

Exercise 3:
Show that the MinCostFlow problem defined on Slide 97 in Chapter 6 can be solved in polynomial time.

Exercise 4:
Assume that all symbols of a search string s are different. Show how to modify the *Simple-Search* algorithm from Slide 7 of Chapter 7 to run in time $O(n)$ on a text t with $|t| = n$.