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Advanced Algorithms

WS 2019

Homework Assignment 1

Problem 1:
A ship enters some harbor and 40 sailors leave it to enjoy themselves. In the night, they all come back, but they are so drunk that each of them picks a random cabin out of the 40 cabins uniformly and independently at random. What is the expected number of sailors that fall asleep in their own cabin?

Problem 2:
Consider a random experiment Ω that can be represented as $\Omega = \Omega_1 \times \Omega_2$ with probability distributions $p_1 : \Omega_1 \to [0,1]$ and $p_2 : \Omega_2 \to [0,1]$ and the property that $\Pr[w] = p_1(w_1) \cdot p_2(w_2)$ for all $w = (w_1, w_2) \in \Omega$. Show that then for any two events $A_1 \subseteq \Omega_1$ and $A_2 \subseteq \Omega_2$ it holds for $A'_1 = A_1 \times \Omega_2$ and $A'_2 = \Omega_1 \times A_2$ that

$$\Pr[A'_1 \cap A'_2] = \Pr[A'_1] \cdot \Pr[A'_2] = \Pr[A_1] \cdot \Pr[A_2] \ .$$

Problem 3:
Show that for any random variable $X : \Omega \to \mathbb{N}$, $\mathbb{E}[X] = \sum_{x \in \mathbb{N}} \Pr[X \geq x]$.

Problem 4:
Prove Theorem 1.5.