Abstract—Letting nodes cooperate improves the performance of wireless networks. To organize this cooperation, many cooperative transmitters employ channel knowledge that they obtain from the destination. Such Channel State Information (CSI) feedback introduces errors and overhead whose degrading effect on the overall performance has not been consistently studied so far. Capturing this degradation, we provide a new framework to analyze cooperation’s outage capacity with limited feedback. Our framework holds for selection relaying and can be easily applied to arbitrary network graphs. Applying it to a simple example shows that selection relaying profits from CSI feedback when the acceptable error rate is high. Surprisingly, no effort for feedback needs to be spent at a strict target error rate.

I. INTRODUCTION

Cooperative relaying has become a well-known approach to improve the performance of wireless networks. By operating as a virtual multi-antenna array, even single antenna nodes obtain spatial diversity gains. Solid analytical work [1, 2] but also recent experiments [3, 4] have shown that these diversity gains can significantly improve the error rate, coverage, energy efficiency, or the throughput of a wireless network.

However, cooperative relaying comes at a cost. First, repeating information reduces the network capacity by a so-called multiplexing loss. Second, protocols add overhead to organize cooperation and often use channel knowledge to decide if and how to forward. This forwarding decision is the key feature of so-called selection relaying protocols which (i) maximize the diversity gain by not forwarding erroneous packets and (ii) minimize the multiplexing loss by forwarding information only when it is needed. The accuracy of this forwarding decision depends on the available channel knowledge. Thus, not only the cost but also the quality of the channel knowledge is critical for the performance of selection relaying.

Typically, cooperative relaying protocols cope with non-reciprocal fading channels. Without reciprocity the transmitter obtains its channel knowledge by Channel State Information (CSI) feedback from the receiver. In practical systems, the capacity of this feedback channel is limited and feedback errors can occur during CSI transmission. Under such practical assumptions, the feedback channel limits the performance that a selection relaying protocol can reach.

So far, this limiting effect of CSI feedback on the performance of selection relaying protocols is only rarely studied in literature. Most authors assume ideal CSI to be available by ignoring feedback overhead and errors. Assuming limited CSI feedback, Lo, Heath, and Vishwanath [5] studied throughput and error rate for a specific path allocation protocol. However, focusing only on average channel gains, the authors ignored feedback errors that were imposed by fading.

Taking fading into account, this paper provides are more general model and analysis. Our model captures the effect of feedback overhead and feedback errors on the outage capacity of cooperative networks. These networks employ conventional selection relaying protocols [1] and can be of arbitrary topology. While this analytical framework holds for any number of relays, we illustrate it for a simple network with two relays. Our numerical results show clearly when cooperative networks with selection relaying profit from CSI feedback. CSI feedback provides substantial outage capacity gains when the target error rate is high. Surprisingly, with a stricter target error rate, selection relaying succeeds without additional CSI. Here, the feedback channel’s error rate dominates the performance of the cooperative networks and simple combining (without feedback) succeeds. In this case, no extra effort for CSI feedback needs to be spent.

Our numerical results can already serve as a look-up table to let hybrid protocols decide when to switch between combining at the receivers and a feedback-based forwarding decision at the transmitters. Results for larger networks and different feedback schemes can be easily obtained with the presented analytical framework.

The paper is structured as follows. In Sec. II we describe selection relaying protocols and classify them according to their CSI demands. By assuming ideal CSI, we jointly analyze all these protocol types in Sec. III. Based on these outage probability and outage capacity results, the effect of limited CSI feedback is modeled and analyzed in Sec. IV. In Sec. V the paper is concluded and future steps are discussed.

II. SELECTION RELAYING PROTOCOLS

This section describes selection relaying protocols. First, we identify common features of protocols from the literature and generalize the operation of selection relaying. Second, we classify selection relaying according to the employed CSI and describe the included cooperation protocols in detail.

A. Selection relaying in general

Two basic forms of selection relaying were described in Laneman’s seminal paper [1]. With Selection Decode-and-
Forward (SDF), a relay does not forward erroneous packets to prevent for error propagation and, thus, reach full diversity. To increase spectral efficiency, Laneman et al. extended SDF by an adaptive forwarding decision that only forwards a packet when requested by the source. This so-called Incremental Relaying (IR) protocol avoids unnecessary retransmissions but relies on CSI feedback.

From these fundamental studies, we can derive two basic principles of selection relaying protocols; both illustrated in Fig. 1.

1) **Regeneration**: The relays decode the received bits and detect erroneously received packets. To this end, it locally extracts receiver CSI (CSI$_{rx}$), e.g., a Cyclic Redundancy Check (CRC) checksum.

2) **Forwarding decision**: Relays decide if and how to forward. This decision can be based solely on CSI$_{rx}$ or can take additional transmitter CSI (CSI$_{tx}$) into account.

Irrespective on the employed CSI, a protocol cycle begins with the source’s initial broadcast to the relays. Then each relay regenerates, obtains its CSI, and performs the forwarding decision (Fig. 1). The cycle ends with a forwarding phase where the relays transmit. As stated in Sec. I, CSI$_{tx}$ is typically obtained via feedback. Using this type of channel knowledge avoids unnecessary retransmissions which substantially decreases the multiplexing loss. Due to this high effect on the performance we classify selection relaying protocols according to their employed CSI. We call the resulting protocol classes Combining-based Selection Relaying (CSR) and Path Selection Relaying (PSR).

**B. Combining-based Selection Relaying protocols**

With these protocols, the cooperative network achieves its diversity gain by combining the directly received and relayed signals at the source. By combining these signals, the destination selects the best network path a posteriori, i.e., after all signals are sent. Coherent combining and the relay’s forwarding decision require only local channel knowledge. Using only this CSI$_{tx}$, a relay forwards irrespective of the state of other parallel links in the cooperative network. For instance, with SDF the relay even forwards if direct transmission has succeeded. Without CSI$_{tx}$, this correct reception cannot be signaled to the relays and the multiplexing loss due to unnecessary retransmissions cannot be avoided.

The CSR protocols listed in Tab. I employ only CSI$_{rx}$ and primarily differ in their coding scheme. While SDF performs repetition coding [1], i.e., the forwarded codeword equals the received codeword, Coded Cooperation (CC) [6] varies the coding rate of the forwarded packets to limit the multiplexing loss. Besides adapting the channel code, a CSR protocol may employ network coding [7, 8] or even space-time coding [9]. In this paper, we will not study such combinations of various coding schemes. Instead, we focus on protocol and feedback operation.

**C. Path Selection Relaying protocols**

If a relay knows the state of other links it can choose not to retransmit if direct transmission succeeds or if a different relay has a better channel state towards the destination. More general, if CSI$_{tx}$ is available, only the nodes on the best end-to-end network path need to transmit to reach full diversity [2]. Naturally, these nodes have to be chosen before they transmit.

Due to this a-priori selection, a PSR protocol avoids the multiplexing loss and, thus, can use the channel more efficiently than CSR. Previous work described many of such protocols. Tab. I lists some relevant examples. These protocols differ in the form of the employed CSI$_{tx}$ and in how this channel knowledge is fed back from the destination to the transmitters. While IR [1] and CoopMAC [10] use explicit feedback from the destination. Many PSR protocols rely on implicit negotiation among the nodes. Typical examples of such protocols are Opportunistic Relaying (OR) [11, 12] and Network Path Selection [2]. In this paper we will not detail these negotiation procedures. Instead, we focus on the effect of limited CSI feedback on the path selection. Note that such limited CSI affects the performance of any PSR protocol irrespective of its particular signaling scheme.

To sum up, depending on their CSI demands, many cooperation protocols fall either in the PSR or in the CSR category. Although intuitive, this categorization has not been used in the literature so far.

**III. OUTAGE ANALYSIS WITH IDEAL CSI**

Before we will study how limited feedback affects PSR, let us jointly analyze PSR and CSR for ideal CSI. After clarifying our basic assumptions and describing the underlying method, we provide outage probability and outage capacity terms for

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arbitrary networks. Based on these terms, numerical examples for a cooperative networks with two relays are provided.

A. Channel and system assumptions

Like many analytic papers on cooperative relaying protocols [1, 2, 6], we assume a channel model with one fading block per relaying cycle. This model implies slow fading where all channel coefficients are assumed to be independent between the cycles and between the nodes. In particular, we model the received signal $y_j$ for a transmission of signal $x_i$ from an arbitrary node $i$ to node $j$ by $y_j = h_{i,j}x_i + n_0$. The random channel coefficients $h_{i,j}$ are i.i.d. Rayleigh-distributed and the additive random noise variable $n_0$ is Gaussian-distributed. Furthermore, we assume non-reciprocal fading which corresponds to $h_{i,j} \neq h_{j,i}$.

The Signal-to-Noise Ratio (SNR) per fading block $\gamma_{i,j} = |h_{i,j}|^2 \Gamma$ is called instantaneous SNR. It depends on the system-wide reference SNR $\Gamma = P_0/N_0$ which represents a scaling factor according to the overall noise power $N_0$ and the system-wide transmit power constraint $P_0$. The mean SNR is $\gamma_{i,j} = \Gamma h_{i,j}$ where the link-dependent factor $\Gamma_{i,j} = d_{i,j}^{-\alpha}$ accounts for the path loss via propagation distance $d_{i,j}$ according to path loss exponent $\alpha$. Note that we normalize $d_{i,j}$ to one unit reference distance with no loss of generality. Unless noted by dB, all SNR values are linear.

At system level, we assume all transmitters to employ the same channel code (i.e., common codebooks) as ideal Shannon code per fading block. This implies that all relays use repetition coding, i.e., forward the received packet with equal code. At all receivers, coherent signal detection is assumed. If the source receives multiple signals, it employs Maximum Ratio Combining (MRC) to achieve diversity gains. Furthermore, we assume perfect error detection which is practical due to the high detection rate of standard CRC codes. A Medium Access Control (MAC) scheme assures that each transmission is performed via an orthogonal channel, perfectly avoiding collisions. With these common assumptions [1, 2], fading is the only error event at high SNR and the outage probability reflects the end-to-end Packet Error Rate (PER).

B. Cut set analysis

Our analysis is based on common graph-theoretical network models and definitions [13, Section 26.1]. We model each network as finite directed graph where each link $(i,j)$ is weighted by its AWGN capacity $C_{i,j} = \log_2(1 + \gamma_{i,j})$. In such flow network each channel’s capacity only depends on the instantaneous SNR $\gamma_{i,j}$. Hence, it suffices to weight each link only by the corresponding instantaneous SNR value. Only links with a positive capacity are included.

Each flow network includes a dedicated source node $a$ and destination node $d$. We assume that each of $N$ relay nodes between $a$ and $d$ lies on some path towards $d$. That is, for any node $r$ there is a path $a \rightarrow r \rightarrow d$. As an example of such flow network, Fig. 2 shows a fully connected network graph with $N = 2$ relays.

![Fig. 2. Fully connected flow network for $N = 2$ relays with the instantaneous SNR $\gamma_{i,j}$ as capacity weight for a link $(i,j)$ and with the unidirectional cut sets $S_1, ..., S_M$.](image)

The figure further includes three cuts illustrated as dashed lines. A cut separates the network into disjoint subsets and a cut set $S_m$ includes all links crossing this cut, e.g., $S_1 = \{(a,c),(a,d),(a,b)\}$ in Fig. 2. The number of links within a cut set $S_m$ is given by the cardinality $|S_m|$ of this cut set. For instance, $S_1$ includes three links and is, thus, of cardinality $|S_1| = 3$. We denote all $M$ cut sets of a flow network by the supersets $S$ with $S := \{S_1, ..., S_M, \ldots, S_M\}$. Note that only unidirectional cut sets are defined in Fig. 2. That is, all links within a cut set cross this set only in a single direction.

C. Outage probability and diversity order

A link $(i,j)$ is in outage if its AWGN capacity $C_{i,j}$ falls below a desired transmission rate $R$ in bits/s/Hz. In case of this outage event $\{C_{i,j}(\gamma_{i,j}) < R\}$, no channel code can fully avoid decoding errors. The outage probability is then defined as the probability $P_\text{out} = P\{C_{i,j}(\gamma_{i,j}) < R\}$. Besides the outage probability we study the diversity order $L$ which represents the number of independent fading blocks per end-to-end transmission. With Rayleigh fading, $L$ exponentially decreases the error rate, i.e., $P_\text{out} \propto (1/\Gamma)^L$. This exponential effect makes the diversity order to an important performance measure at high SNR. Let us now employ cut-set analysis to derive $L$ and $P_\text{out}$.

With cooperative relaying, multiple links are employed in parallel and these links are included in $M$ cut sets. Given all cut sets $S$, we can find the diversity order $L$ by searching the cut sets

$$S_M := \{S \in S \mid |S| = L\}$$

that include the minimum number of links

$$L = \min_{S \in S}|S|.$$

This smallest cardinality over all its cut sets, is the diversity order $L$ of the network [14]. The rationale behind this definition is that $L$ represents the number of independent links which at least have to fail to cause the overall end-to-end transmission to be in outage. This “bottleneck” is given by the cut set of smallest cardinality.

The outage probability for arbitrary flow networks with selection relaying at high SNR is given in [14] as

$$P_\text{out} \approx \frac{1}{L!} \Theta \left(\frac{2KR - 1}{\Gamma}\right)^L.$$


Beside the transmitter rate $R$ and reference SNR $\Gamma$, (3) depends on the number of employed orthogonal channel $K$ and the exponent $L$.

Equation 3 further includes the link-dependent term
\[
\Theta = \sum_{\forall S_m \in S_M} \left( \prod_{(i,j) \in S_m} \frac{1}{\Gamma_{i,j}} \right)
\]  
(4)

where we define the $M$ cut sets $S_M \subseteq S$ of minimal cardinality $L$ as in (1).

While deriving (3) and (4) Boyer et al.
made use of the fact that, given common codebooks, the end-to-end outage probability is upper bounded by the outage probability of the cut sets $S_M$ [14]. Put less formally, no cut set with more than $L$ links can decrease the overall $P_{\text{out}}$ below the outage probability given by this “bottleneck”. Therefore, (4) accounts only for the links of those cut sets $S_M$ that define the diversity order $L$.

### D. Outage capacity for arbitrary networks

The outage capacity $C_{\text{out}}$ is defined as the highest data rate such that a given outage probability constraint $\epsilon$ is not exceeded [15, 5.4.1]. Practically speaking, $C_{\text{out}}$ measures the maximum data rate guaranteed for at least $(1-\epsilon)\cdot 100\%$ of the time. Thus, $\epsilon$ reflects a target error rate which is an important design parameter of many wireless systems [16].

With this error rate constraint, we can obtain the outage capacity
\[
C_{\text{out}} := \sup \{ R : P_{\text{out}}(R) \leq \epsilon \}
\]  
(5)

by solving $P_{\text{out}}(R) = \epsilon$ for $R$. Applying this to (3) results in
\[
C_{\text{out}} := R = \frac{1}{K} \log_2 \left( 1 + \Gamma \frac{L \epsilon}{\Theta} \right) \text{[bits/s/Hz]}
\]  
(6)

as the end-to-end outage capacity at high SNR for arbitrary cooperative networks with selection relaying. It should be noted that for any feasible value of $\epsilon$, $L$, and $\Theta$, the term $L \epsilon / \Theta$ in (3) is non-negative and, hence, a real-valued solution of $C_{\text{out}}$ can be obtained.

In (6), the outage capacity is linearly reduced by the multiplexing loss $1/K$. This clearly expresses the costs of retransmitting information via orthogonal channels. With relaying, $K > 1$ nodes may transmit per end-to-end transmission. With repetition coding, each transmitter equally employs one subchannel which requires to split the end-to-end capacity by $K$.

### E. Numerical results and discussion

To illustrate the above method, we apply (3), (4), and (6) on the network in Fig. 2. The results for this fully connected network with $N = 2$ relays are shown in Fig. 3. We assume a path loss exponent of $\alpha = 2.4$ and a symmetric “diamond” geometry. Here, all node-to-node distances are 1 unit except for the direct link where the geometry requires $d_{a,d} = \sqrt{2}$. To account for networks with low and high target error rate, we study two levels of $\epsilon$.

To highlight the effect of $\epsilon$, $\Theta$, and $K$, we plot $C_{\text{out}}$ as a fraction of the AWGN capacity $C_\text{i} = \log_2(L \Gamma)$. With this strict but also common normalization [17], $L$ corresponds to the diversity order of the studied relaying scheme.

Due to the three links in $S_1$ and $S_3$ (Fig. 2), PSR and CSR reaches the diversity order $L = 3$; cp. (2). With CSR, source and both relays transmit which leads to $K = 3$ subchannel uses per cycle. By choosing either direct transmission, path $a \rightarrow c \rightarrow d$, or path $a \rightarrow b \rightarrow d$, PSR employs only $K = 2/3$ subchannels on the average. Note that PSR never chooses path $a \rightarrow b \rightarrow c \rightarrow d$ since this path reaches only a diversity order of two at node $c$.

In Fig. 3, we compare the results for cooperative relaying to Non-Cooperative Relaying (NCR) via path $a \rightarrow c \rightarrow d$ and to direct transmission. Naturally, direct transmission achieves $L = 1$ at $K = 1$ while NCR requires $K = 2$ subchannel uses for reaching merely $L = 1$.

For these cases, Fig. 3(a) shows the outage capacity at $\epsilon = 10^{-3}$. With this low target error rate, direct transmission
and NCR perform poorly for the complete SNR region. Mutually comparing the protocol classes clearly shows that, at low \( \varepsilon \), PSR outperforms CSR when the SNR is high. Here, PSR employs the channel more efficiently than CSR by relaying a signal only when needed. With decreasing SNR the situation reverses. Here, CSR performs best since it maximizes the diversity gain by using all available links toward \( d \). Consequently, with strict error rate constraints and medium or low SNR, CSR protocols are preferable.

Fig. 3(b) with \( \varepsilon = 10^{-1} \) represents a typical error rate acceptable for non-real time traffic [16]. At this high \( \varepsilon \), even direct transmission shows its benefits. Transmitting directly achieves 50\% of the AWGN capacity at \( \Gamma = 31 \) dB and, thus, outperforms any relaying scheme at high SNR. Here, PSR chooses the direct path to \( d \) which is represented by the sharp bend of \( C_{\text{out}} \) at 23 dB. At low SNR, PSR outperforms all other relaying protocols for all studied SNR levels. Hence, PSR seems to be the preferred protocol choice at high target error rates. However, we will see that this simple conclusion only holds at a first glance.

IV. OUTAGE CAPACITY WITH LIMITED CSI FEEDBACK

So far, we assumed perfect channel knowledge to be available at the transmitters at no costs. Utilizing this ideal CSI allowed PSR to clearly outperform CSR. Nonetheless, in practical systems feedback errors and overhead limit CSI. To capture this limitation, we describe a new analytical model in this section. Then, we use this model to study the outage capacity and region of operation for a simple cooperative network. Our results show that with limited feedback, CSR still outperforms PSR at low SNR.

A. Outage capacity for arbitrary networks

Our basic modeling idea is to condition PSR’s outage capacity \( C_{\text{out}} \) on the outage capacity that is employed for feedback. Put formally, we write

\[
C_{\text{out}}^{\text{PSR,FB}} = R_{\text{FB}}(b_{\text{FB}}, N_T) \cdot C_{\text{out}}^{\text{FB}}
\]

where \( R_{\text{FB}} \) is the share of the feedback channel’s outage capacity \( C_{\text{out}}^{\text{FB}} \) that remains after correct CSI transmission. Assuming that \( b_{\text{FB}} \) bits of CSI are transferred once per feedback period of \( N_T \) protocol cycles, we can define this share as

\[
R_{\text{FB}}(b_{\text{FB}}, N_T) := \begin{cases} \frac{C_{\text{out}}^{\text{FB}} - b_{\text{FB}}}{N_T} ; & b_{\text{FB}} / N_T \leq C_{\text{out}}^{\text{FB}} \\ 0 ; & \text{otherwise} \end{cases}
\]

Note that this definition captures the feedback overhead by \( b_{\text{FB}} \), feedback frequency by \( 1 / N_T \) as well as the feedback channel’s capacity and error constraints by \( C_{\text{out}}^{\text{FB}} \).

Naturally, this simple model has several limitations. First, setting the lower case in (8) to zero implies that a PSR protocol gives up if no CSI feedback transmission is possible, i.e., if \( b_{\text{FB}} / N_T > C_{\text{out}}^{\text{FB}} \). Consequently, (8) and (7) are zero. In realistic slow fading channels, a practical PSR protocol would reach \( C_{\text{out}}^{\text{PSR,FB}} > 0 \) by using previously received CSI. We can account for such fall-back option by setting the lower case in (8) to some empirical value (obtained by simulation or experiment). However, for our study below, let us employ (8) as a pessimistic model.

Second, we assumed that the target outage probability for data \( \varepsilon_{\text{Data}} \) is equal to the target outage probability for CSI feedback \( \varepsilon_{\text{FB}} \). One can easily insert different values for \( \varepsilon_{\text{Data}} \) and \( \varepsilon_{\text{FB}} \) in (8) and (7). Due to the high relevance of CSI feedback, usually \( \varepsilon_{\text{FB}} \leq \varepsilon_{\text{Data}} \). Nonetheless, we use \( \varepsilon := \varepsilon_{\text{FB}} \) for simplicity.

Finally, using \( C_{\text{out}}^{\text{FB}} \) in (8) implies a specific feedback procedure. In our study below, we assume that \( d \) transmits its CSI via a single broadcast to all transmitters. This simple form of feedback adds only a single subchannel use \( K \) and is, thus, very efficient in terms of channel uses. We can easily derive the outage capacity of this feedback channel by applying (4), (3), and (6) as above. This yields

\[
C_{\text{out}}^{\text{FB}} = \log_2(\varepsilon \Gamma_{d,a} \Gamma_{d,b} \Gamma_{d,c} \Gamma + 1)
\]

which captures a variety of proactive and reactive OR protocols [2, 11, 12] where either the source or all relays rely on CSI to choose the single best path towards \( d \).

B. Choosing feedback parameters

Before applying the above model to obtain numerical results, we have to choose a number of CSI feedback bits \( b_{\text{FB}} \) and the feedback period \( N_T \). Choosing \( b_{\text{FB}} \) depends on the required CSI accuracy and the number of paths. With \( N \) relays and the direct link, \( d \) selects the best out of \( N + 1 \) paths such that \( C_{\text{out}} \) is maximized. To signal this selection to the transmitters, \( b_{\text{FB}} = \log_2(N + 1) \) bits have to be transferred. In Fig. 2, this leads to \( b_{\text{FB}} = \log_2 3 \) bits that \( d \) transmits once every \( N_T \) cycles.

Choosing this feedback period depends on the coherence time of the fading channel. To perfectly synchronize PSR’s operation to a block fading channel, CSI feedback is required once per fading block. As we assumed one block per MAC cycle, this case is expressed by \( N_T = 1 \), i.e., one feedback transmission per cycle. The more practical case, however, is limited CSI with occasional feedback. Here, \( N_T > 1 \) can be chosen if the channel’s coherence time is longer than the MAC cycle time \( T_{\text{cycle}} \). For instance, assuming a typical \( T_{\text{cycle}} = 1 \) ms [16], a slow walking speed of 1 m/s, and 2.6 GHz carrier frequency leads to an approximate channel coherence time of 26 ms [18]. In this system, we can synchronize CSI to the channel by updating feedback once every \( N_T = 26 \) protocol cycles.

Naturally, more frequent feedback is required with faster nodes. Furthermore, \( b_{\text{FB}} \) increases with more sophisticated forms of channel adaptation, e.g., if \( d \) also assigns the transmission rate to the relays. However, let us now demonstrate the above methods for \( N_T = 26 \) and \( b_{\text{FB}} = \log_2 3 \) bits.

C. Numerical results and discussion

In Fig. 4 the outage capacity for PSR with several degrees of CSI is shown. PSR with ideal CSI and NCR without CSI represents the upper and lower bound, respectively. As
protocols require additional resources to protect their CSItx, e.g., by cooperating even during CSI feedback.

For high target error rate, this situation reverses. Fig. 4(b) shows the above protocols and CSI degrees for $\varepsilon = 10^{-1}$. At such high $\varepsilon$ the full CSI case is only slightly degraded by feedback errors. Here, a single broadcast channel provides sufficient capacity to transfer the feedback information. Consequently, even if we account for overhead and feedback errors, CSR protocols are significantly outperformed by PSR when the acceptable error rate is high.

D. Region of operation

The above results show that choosing the best relaying protocol to maximize $C_{\text{out}}$ highly depends on the available CSI, the outage probability constraint $\varepsilon$, and on the SNR regime. Depending on these parameters, the $C_{\text{out}}$ functions intersect, making either PSR or CSR a good choice. This region of operation where on protocol class outperforms the other is summarized in Fig. 5. For various $\varepsilon$, the figure shows the reference SNR value $\Gamma$ where $C_{\text{out}}^A = C_{\text{out}}^B$. If $\Gamma$ increases above the plotted line, $C_{\text{out}}^A$ exceeds $C_{\text{out}}^B$. Hence, for an SNR above a shown line, case $A$ is preferable while, below the line, case $B$ achieves higher capacity.

We highlight the resulting regions of operations by different colors in Fig. 5. Since direct transmission always requires largest SNR, it is only feasible at very high SNR or at high $\varepsilon$. When the target error rate decreases below $10\%$, CSR outperforms all other cases for a reasonable SNR region.

PSR reaches its region of operation between CSR and direct transmission. Taking limited CSI feedback into account reduces PSR’s region of operation in favor of CSR. This reduction is highest at low $\varepsilon$ and, as discussed above, results from the low diversity order of direct feedback.

All in all, Fig. 5 allows to choose the relaying protocol and network that maximizes the outage capacity at a given target error rate and expected SNR. Our results can also serve as a look-up table to dynamically choose the relaying protocol according to the measured SNR.

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(a) Outage capacity as a fraction of AWGN capacity vs. reference SNR for $\varepsilon = 10^{-3}$.

(b) Outage capacity as a fraction of AWGN capacity vs. reference SNR for $\varepsilon = 10^{-1}$.

Fig. 4. Outage capacity as a fraction of AWGN capacity: Numerical results with ideal and limited CSI shown for two levels of $\varepsilon$.

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Fig. 5. Region of operation: Reference SNR at intersection of the two capacity functions $C_{\text{out}}^A = C_{\text{out}}^B$. Numerical results shown vs. $\varepsilon$ for $\alpha = 2.4$. 

V. CONCLUSION AND FUTURE RESEARCH

Starting with an overview of cooperative relaying protocols, we compared two types of selection relaying: (i) CSR which employs only receiver CSI for its forwarding decision and (ii) PSR which employs transmitter CSI that is obtained via feedback. To capture the effect of limited CSI feedback, we described a new analytical framework that conditions the outage capacity of the cooperative transmission on the outage capacity of the feedback channel. Based on cut set analysis, the presented methods hold for arbitrary cooperative networks with selection relaying protocols. This framework is general, easy to use, and, thus, is a large step in understanding cooperative networks under practical feedback constraints.

Applying this new framework shows when cooperative networks with selection relaying profit from CSI feedback. At high SNR and at high target error rate, feedback enables a selection relaying protocol to save retransmissions. By decreasing the multiplexing loss, the additional channel knowledge improves the network’s capacity even if feedback transmission errors and overhead are taken into account. When SNR or target error rate decrease, the feedback channel’s error rate starts to dominate the outage capacity of the cooperative network. With decreasing target error rate or SNR, more and more capacity has to be spend for accurate CSI feedback until, finally, obtaining additional channel knowledge via feedback is not worth the effort. Here, CSR cooperation protocols succeed by employing only local CSI.

These conclusions provide clear guidelines to design future cooperation protocols. Depending on the current region of operation, hybrid selection relaying protocols may adapt the number of combined signals and may switch between a local and a feedback-based forwarding decision. To perform such adaptation, our numerical results can be used as a look-up table. Results for larger networks can be easily obtained with our analytical framework. It is our hope that these simple methods motivate the research community to study cooperative relaying more rigorously under practical feedback assumptions.

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